LinesPlanes

For Exercises 1-4, write the line $L$ through the point $P$ and parallel to the vector $v$ in the following forms: (a) vector, (b) parametric, and (c) symmetric.

1) $P = (2, 3, -2), v = (5, 4, -3)$

2) $P = (3, -1, 2), v = (2, 8, 1)$

3) $P = (2, 1, 3), v = (1, 0, 1)$

4) $P = (0, 0, 0), v = (7, 2, -10)$

PROBLEM 2:

```python
var ('p,v,t,s,u,q,L,x,y,z,r')
p = vector([3,-1,2])
v = vector([2,8,1])
# vector form:
L = p+t*v
L
(2*t + 3, 8*t - 1, t + 2)
```

```python
r = vector([x,y,z])
e = r-L
solve(e[0],t); solve(e[1],t); solve(e[2],t)
[t == 1/2*x - 3/2]
[t == 1/8*y + 1/8]
[t == z - 2]
```

PROBLEM 4:

```python
L = t*vector([7,2,-10])
L
(7*t, 2*t, -10*t)
```

```python
e = r-L
[solve(e[i],t) for i in range(len(e))]
[[t == 1/7*x], [t == 1/2*y], [t == -1/10*z]]
```

For Exercises 5-6, write the line $L$ through the points $P_1$ and $P_2$ in parametric form.

5) $P_1 = (1, -2, -3), P_2 = (3, 5, 5)$
6) \( P_1 = (4, 1, 5), P_2 = (-2, 1, 3) \)

**PROBLEM 6:**

```python
p1 = vector([4, 1, 5])
p2 = vector([-2, 1, 3])
L = p1 + t*(p2-p1)
L
```

\((-6t + 4, 1, -2t + 5)\)

For Exercises 7-8, find the distance \( d \) from the point \( P \) to the line \( L \).

7) \( P = (1, -1, -1), L: x = -2 - 2t, y = 4t, z = 7 + t \)

8) \( P = (0, 0, 0), L: x = 3 + 2t, y = 4 + 3t, z = 5 + 4t \)

**PROBLEM 8:**

```python
L = vector([3+2*t, 4+3*t, 5+4*t])
p = vector([0, 0, 0])
q = vector([3, 4, 5])
v = vector([2, 3, 4])
d = (p-q).cross_product(v).norm()/v.norm()
d; d.N()
```

\(1/29*sqrt(6)*sqrt(29)\)

\(0.454858826147342\)

For Exercises 9-10, find the point of intersection (if any) of the given lines.

9) \( x = 7 + 3s, y = -4 - 3s, z = -7 - 5s \) and \( x = 1 + 6t, y = 2 + t, z = 3 - 2t \)

10) \( \frac{x - 6}{4} = y + 3 = z \) and \( \frac{x - 11}{3} = \frac{y - 14}{-6} = \frac{z + 9}{2} \)

```python
# problem 10.
solve([6+4*t == 11+3*s, -3+t==14-6*s, t==9+2*s],s,t)
[]

NO SOLUTION!
```

# problem 9.
\[
\text{solve(}\left[\begin{array}{c}7+3s=1+6t,\ -4-3s=2+t,\ -7-5s=3-2t\end{array}\right],s,t)\\
\left[\begin{array}{c}s=-2,\ t=0\end{array}\right]
\]
\[
e=\left[\begin{array}{c}7+3s=1+6t,\ -4-3s=2+t,\ -7-5s=3-2t\end{array}\right]\\
[e[i].\text{substitute}(s=-2,t=0)\ \text{for}\ i\ \text{in}\ \text{range}(\text{len}(e))]\\
[1==1,\ 2==2,\ 3==3]
\]

For Exercises 11-12, write the normal form of the plane \(P\) containing the point \(Q\) and perpendicular to the vector \(n\).

11) \(Q=(5,1,-2),\ n=(4,-4,3)\)

12) \(Q=(6,-2,0),\ n=(2,6,4)\)

```python
# problem 12.
q = vector([6,-2,0])
n = vector([2,6,4])
r = vector([x,y,z])
n.dot_product(r)-q.dot_product(n) == 0
2*x + 6*y + 4*z == 0
```

```python
# problem 11.
q = vector([5,1,-2])
n = vector([4,-4,3])
r = vector([x,y,z])
n.dot_product(r)-q.dot_product(n) == 0
4*x - 4*y + 3*z - 10 == 0
```

For Exercises 13-14, write the normal form of the plane containing the given points.

13) \((1,0,3),(1,2,-1),(6,1,6)\)

14) \((-3,1,-3),(4,-4,3),(0,0,1)\)

```python
# problem 14.
p = vector([-3,1,-3])
q = vector([4,-4,3])
r = vector([0,0,1])
n = (q-p).cross_product(r-p)
n.dot_product(vector([x,y,z])) - p.dot_product(n) == 0
-14*x - 10*y + 8*z - 8 == 0
```

15) Write the normal form of the plane containing the lines from Exercise 9.
16) Write the normal form of the plane containing the lines from Exercise 10

```python
# problem 16.
# There is no such plane. The two lines are skew lines.

# problem 15.
# The plane goes through (1,2,3) and has directions v1 and v2.

p = vector([1,2,3])
v1 = vector([3,-3,5])
v2 = vector([6,1,-2])
P = p + s*v1 + t*v2
P
(3*s + 6*t + 1, -3*s + t + 2, 5*s - 2*t + 3)
```

# the normal equation for this plane is...

```python
n = v1.cross_product(v2)
r = vector([x,y,z])
n.dot_product(r) - p.dot_product(n) == 0
```

\[ x + 36*y + 21*z - 136 == 0 \]

For Exercises 17-18, find the distance \( D \) from the point \( Q \) to the plane \( P \).

17) \( Q = (4,1,2) \), \( P: 3x - y - 5z + 8 = 0 \)
18) \( Q = (0,2,0) \), \( P: -5x + 2y - 7z + 1 = 0 \)

```python
# p39/18
q = vector([0,2,0])
n = vector([-5,2,-7])
p = vector([0,-1/2,0])

d = (q-p).dot_product(n).norm()/n.norm()
d; d.N()
```

\[ 5/78*sqrt(78) \]

\[ 0.566138517072298 \]

For Exercises 19-20, find the line of intersection (if any) of the given planes.

19) \( x + 3y + 2z - 6 = 0 \), \( 2x - y + z + 2 = 0 \)
20) \( 3x + y - 5z = 0 \), \( x + 2y + z + 4 = 0 \)

```python
# p39/20
n1 = vector([3,1,-5])
```
n2 = vector([1,2,1])

v = n1.cross_product(n2)
# when z=0 we have the common point
solve([3*x+y==0, x+2*y==-4],x,y)

[[x == (4/5), y == (-12/5)]]

p = vector([4/5,-12/5,0])
L = p + t*v
L

(11*t + 4/5, -8*t - 12/5, 5*t)

PROBLEM 21.

Find the point(s) of intersection (if any) of the line \( \frac{x-6}{4} = y+3 = z \) with the plane \( x + 3y + 2z - 6 = 0 \). (Hint: Put the equations of the line into the equation of the plane.)

solve([(x-6)/4 == y+3, (x-6)/4 == z, y+3 == z, x+3*y+2*z-6 == 0],x,y,z)

[[x == 10, y == -2, z == 1]]