Maximum-Entropy Velocity Profiles and Boundary Layer Theory in Laminar to Turbulent Flow

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Abstract

This study applies Jaynes’ MaxEnt method \[1\] - as extended by Chiu \[2\] - to determine the “most probable” steady-state velocity profile \(u(y)\) in several classical fluid flow systems, including axial flow in a cylindrical pipe \[2\] and flow between parallel plates. In each case, the analysis yields an \textit{analytical solution} for the velocity profile over the entire range of laminar to turbulent flow, as a function of the maximum velocity \(u_{\text{max}}\) and parameter \(M\). In each case, the predicted profile reduces to the well-known laminar solution at \(M = 0\), whilst for \(M > 0\) it gives an equation which supersedes the semi-empirical correlations commonly used for turbulent flow profiles \[2-4\]. For plane parallel flow, to match the known solutions at \(M = 0\) it is necessary to consider the relative entropy function (Kullback-Leibler divergence), which incorporates the Bayesian prior distribution. The main steps of the analysis - including handling of the prior - and the predicted profiles are presented here.

The analysis is then used to derive a new maximum-entropy laminar-turbulent boundary layer theory, for the velocity profile in steady flow along a flat plate. For \(M = 0\), this approximates the Prandtl-Blasius solution for laminar boundary layer flow \[4,5\]. For turbulent flow, it yields a previously unreported solution set.

References:


Key Words: MaxEnt; fluid mechanics; velocity profile; turbulent flow; boundary layer.