LESSONS ABOUT LIKELIHOOD FUNCTIONS FROM NUCLEAR PHYSICS

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Abstract

During the last half century of nuclear-physics research, numerous basic physical quantities have been measured many times. It is instructive to review the historical records of these measurements with the goal of determining how well repeated measurements of the same quantity compare. In most data analyses, the normal (Gaussian) distribution is assumed to appropriately characterize the likelihood, that is, the conditional probability of each measurement, given an assumed value, $p(\text{measurement} \,|\, \text{assumed value})$. On the other hand, there is ample evidence in nuclear physics of significant disagreements among measurements, which are inconsistent with the normal distribution, given their stated uncertainties. Equivalently stated, chi-squared for the best fit to data sets can be much larger than the number of data points. Clearly, outliers exist. To properly describe the properties of uncertainties in real measurements, likelihood functions must allow for larger deviations than described by the normal distribution.

The purpose of the present study is to characterize the likelihood function for real experimental measurements. The histories of measurements for a variety nuclear-physics experiments are examined to determine what can be inferred about the distribution of their values relative to their stated uncertainties. In the present analysis, the uncertainties that arise from independent, random fluctuations and systematic effects are considered separately. Systematic uncertainties typically arise from estimated corrections to the data, such as detector efficiencies and deadtimes, and absolute measurements, such as target densities and integrated beam fluence. These kinds of uncertainties are often difficult for experimenters to estimate. They may be the most likely cause for seriously underestimating the experimental uncertainty.

The historical record provides evidence that experimenters tend to underestimate the systematic uncertainties in their results. Furthermore, experimenters sometimes seem to avoid serious disagreement with previous measurements. Some implications for Bayesian analysis are drawn.

Key Words: likelihood, normal distribution, non-normal distribution, long-tailed likelihood functions, measurements, systematic uncertainty, inconsistent data, outliers