Inverse Langevin approach to time-series data analysis

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Outline

1 Motivation
   - Brownian Motion
   - Limit from discrete process
   - Inference about forces/noise

2 Parabolic scheme for stochastic inference
   - Newton’s method
   - Implementation
   - Examples
   - A very naive application to finance
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Classic Brownian motion

- Einstein: drunkard walk (Smoluchowski controversy)
- Lots of “Brownian” motions all around: physics, finance, communication etc.
- Random Gaussian increments (SDE’s — Wanier, Itô)

\[ dx = m(x; t) \, dt + \sigma(x; t) \, dB \]
Langevin dynamics

- We must retain Newtonian physics:
  - expand the state

\[
dv = a(x, v; t) \, dt + D(x, v; t) \, dB \\
\]
\[
dx = v \, dt \\
\]

- rich set of solutions from simple equations: linear, exponential, oscillatory, etc

- Similar results (classic Brownian)
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Limit from discrete process

- What we tackle (work in progress)
  - Wanier noise $dB = \beta \sqrt{dt}$: Fokker-Planck/forward Kolmogorov equation. (Brownian motion)
  - Non-analytic noises, Levy, and others: Kramer-Moyal equation. (gas of rigid spheres)
Questions

😊 What the drift and the noise?
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- 😞 Is it Fokker-Planck or Kramer-Moyal?
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- 😊 What is the drift and the noise?
- 😞 Is it Fokker-Planck or Kramer-Moyal?
- 😞 Can we compare Fokker-Planck to Kramer-Moyal models?
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**Motivation**
Parabolic scheme for stochastic inference

**Summary**
Brownian Motion
Limit from discrete process
Inference about forces/noise

---

**Markov Chain**

\[
\begin{align*}
y_0 & \rightarrow y_1 & \rightarrow y_2 & \rightarrow \ldots & \rightarrow y_N \\
W_0 & \rightarrow W_1 & \rightarrow W_2 & \rightarrow \ldots & \rightarrow W_N \\
\theta, \eta &
\end{align*}
\]
Use Bayes theorem...

- Likelihood ($\mu$: measurement noise, $\vec{w}$: hidden state, $\eta, \theta$: parameters of interest)

$$p(\eta|\vec{y}) = \frac{1}{p(\vec{y})} p(\eta)p(\vec{y}|\eta)$$

$$p(\vec{y}|\eta) = \int d\vec{w} d\mu d\theta p(\eta, \theta)p(\mu)p(\vec{y}|\vec{w}, \mu)p(\vec{w}|\eta, \theta)$$

- Similar thing for $p(\theta|\vec{y})$
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Exactly soluble propagators

- A Markov process is characterized by its propagator

\[ p(w_0, w_1, \ldots, w_N) = p(w_0)p(w_1|w_0) \ldots p(w_N|w_{N-1}) \]

- We know analytical (Gaussians!) results only for simplest cases

\[ \frac{F(x, v; t)}{m} = -\gamma v - \omega^2 x + a_0 \]
The parabolic method

- We don’t know how to integrate every model
- Newton approximation
  - The force at a small $\delta t$ is almost constant (but depends on initial position + parameters)
  - We can calculate the trajectory/propagator
  - Repeat this procedure for the next step
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Delta function approximation

- Conjugate prior for $p(\mu)$. At some level approximation for large data sets...

$$
\int d\mu \, p(\mu)p(\vec{y}|\vec{w}, \mu) \sim \delta(\vec{y} - \vec{x})
$$

- Now we have only Gaussian integrations over velocities.

$$
\Phi = \int d\vec{v} \, p(y_0, v_0, y_1, v_1, \ldots, y_N, v_N|\theta, \eta)
$$
Main loop

- At each integration step, collect a bunch of coefficients

\[
\eta^\alpha_i \frac{e^{-\frac{\eta}{2} (a_i v_i^2 + 2b_i v_i + 2c_i v_{i-1} + d_i v_{i-1}^2 + 2e_i v_{i-1} + f_i)}}{G_i}
\]

- After each step, some coefficients must be updated before we start with the next integration

\[
\Phi = \int d\vec{v} \ p(\vec{y}, \vec{v}|\theta, \eta) = \frac{\eta^{N-1}}{G(\theta)} e^{-\frac{\eta}{2} f(\theta, \vec{y})}
\]
Inference results

- Use Laplace approximation to normalize \( p(\theta|\bar{y}, \eta) \), plug-in a Gamma prior \( p(\eta) \)

\[
p(\theta|\bar{y}) \propto \frac{p(\theta)}{G(\theta) \left[ f(\bar{y}) + f(\theta, \bar{y}) - f(\bar{\theta}, \bar{y}) + \delta \right]^{\frac{N+1}{2}} + \sigma + \frac{d}{2}}
\]

- \( p(\eta|\bar{y}) \) is calculated analytically
Inference results

- Use Laplace approximation to normalize $p(\theta|\vec{y}, \eta)$, plug-in a Gamma prior $p(\eta)$

$$
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$$

- $p(\eta|\vec{y})$ is calculated analytically

- 🍃 We use a flat prior $p(\theta)$ (I’m ashamed!)
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Oscillatory brownian motion

![Graph showing force parameters vs. data length]

- Constant
- Linear
- Quadratic

Data length
-0.20
-0.15
-0.10
-0.05
0.00
0.05
Coefficient
-0.05
-0.10
-0.15
-0.20

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Comments

- Works pretty well with artificially generated time series
- Still, it does not get the diffusion coefficient quite right in some cases
- It is also somewhat robust to the existence of dissipative forces
Some results

<table>
<thead>
<tr>
<th></th>
<th>$\eta(1)$</th>
<th>$\theta_1(0)$</th>
<th>$\theta_2(0.1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>0.96(0.03)</td>
<td>-0.07(0.02)</td>
<td>$-9 \times 10^{-7}(5 \times 10^{-4})$</td>
</tr>
<tr>
<td>Harmonic</td>
<td>0.78(0.03)</td>
<td>-0.04(0.08)</td>
<td>$-0.099(0.003)$</td>
</tr>
<tr>
<td>Dissipative</td>
<td>1.06(0.03)</td>
<td>0.08(0.02)</td>
<td>$-5 \times 10^{-4}(9 \times 10^{-4})$</td>
</tr>
<tr>
<td>D-H</td>
<td>1.07(0.03)</td>
<td>-0.04(0.08)</td>
<td>$-0.096(0.004)$</td>
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</tbody>
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You will not win any money!

There is no force. **This will NOT make you win money!**
You will not win any money!

- There is no force. **This will NOT make you win money!**

- But this is the expected behavior, of course

- Things to explore (speculation-crash patterns):
  - noise may not be Wanier
  - volatility time
  - we can introduce trends as a time-dependent force
The time series data

GE closing values

-1.5
-1.0
-0.5
0.0
0.5
1.0

-1.5
-1.0
-0.5
0.0
0.5
1.0

log2-return

GE closing values


year
Summary

Given $f(x, \theta)$, it is possible to infer $\theta$ and the intensity of the noise which governs a Langevin system.
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- Open possibilities: better linear approximations, time-dependent parameters, linear diffusion coefficient.
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- Open possibilities: better linear approximations, time-dependent parameters, linear diffusion coefficient.

- Open issues
  - Work out a decent prior $p(\theta)$ (and calculate the evidence!)
  - Treatment of noise is not really satisfactory
  - Dissipative forces (easy) and non-Waenier and Levy noises (potentially very hard)